Black Hole Thermodynamics

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Despite the seemingly impossible task of looking at a black hole experimentally, the thermodynamics of black holes is still a active field of theoretical study. In an effort to reconcile our current understanding of the nature of black holes with the laws of thermodynamics, we encounter black hole entropy as a consequence of the preservation of said laws. By postulating that a black hole must have entropy, we are able to develop four laws that describe the specific nature of the thermodynamics of black holes. Furthermore, we can examine black holes over both finite and infinite amounts of time in such a way that we can investigate how black holes behave as potential heat engines.

Framework for Black Hole Thermodynamics

While black hole thermodynamics is still a relatively new field of theoretical physics, the underlying concepts that have driven the field began with the advent of modern thermodynamics and the development of the now-well-understood Laws of Thermodyanmics (herein classical thermodynamics). Prior to the work of Stephen Hawking and Jacob Beckenstein in the 1970s and beyond, it was not understood if black holes could be described by their temperature as the black hole event horizon was thought to prevent blackbody radiation from being emitted. In 1970, Stephen Hawking developed his "Horizon Area Theorem," which stated that

Horizon Area Theorem. *The total black hole event horizon area in a closed system containing black holes can only increase or stay the same.*

Jacob Beckenstein went on to rectify Hawking's Horizon Area Theorem with the second law of classical thermodyanmics, which states that for an isolated system, the total entropy will only increase or stay the same. What Beckenstein developed was a rather fundamental statement to the field of black hole thermodynamics, which said that "the total area of a closed system never decreases." The issue with this phrase was that it begs the question, are black holes an exception to the laws of thermodynamics as we understand them? The answer was not understood in the early 70s, as it could be possible to violate the second law of thermodynamics by throwing mass into a black hole, provided the black hole did not possess any entropy of its own. Doing so would decrease the total entropy of the system, as there would be a loss of information when the black hole "ate" the entropy of the object as it fell behind the event horizon. To fix this and satisfy the second law of thermodynamics, Beckenstein postulated that black holes have entropy of their own. This combined with Hawking's Horizon Area Theorem tells us that the surface area of a black hole event horizon is necessarily a manifestation of its entropy. Beckenstein's postulation allowed Hawking to assert that if a black hole has entropy, then it must have temperature, and if it has temperature, then it must radiate like a black body. It wasn't until 1974 that Hawking was able to answer the question of how a black hole could radiate if nothing can escape the event horizon. Following the work of Beckenstein, Hawking showed that as black holes have entropy proportional to their event horizon area, they must radiate through quantum vacuum fluctuations. These vacuum fluctuations near event horizons are known as "Hawking Radiation." With this, black holes can be treated as an ideal black body, which tells us that Hawking radiation must necessarily be emitted with the same spectrum as an ideal black body. The issue being that the temperature of a black hole is so low that it's essentially impossible to observe, which explains why black holes look so "cold." For a solar-mass black hole, we can expect the black body spectrum to look like that of something with a temperature on the order of 100 nanoKelvin. For increasingly massive black holes, the temperature only decreases from there. And so we see that black hole temperature is inversely proportional to its mass. An interesting side effect of this perscription for black hole temperature is that it is possible for a black hole to "evaporate" away. That is to say, via Hawking radiation, it is possible for a black hole to lose mass very slowly over time, such that over a very long time (on the order of billions to trillions of years), a black hole could simply disappear from existence. As black hole temperature is inversely proportional to mass, Hawking radiation is emitted at higher rates where tidal forces are stronger, such that evaporation time T_E is proportional to the cube of its mass

$$T_E \propto M^3 \tag{1}$$

An unfortunate caveat of this relationship to all observational astrophysicists is that in order for a black hole to evaporate, it would need to have a mass on the order of the moon, which would place it firmly around the size of a particle of smoke with a temperature of approximately 2.7 Kelvin. This mass is necessary because anything more massive would have a temperature below 2.7 Kelvin, the temperature of cosmic microwave background (CMB) radiation. For a black hole that is colder than the CMB, we would observe a net energy flow into the black hole via CMB radiation, thereby increasing its mass and further decreasing it's temperature. For a black hole hotter than 2.7 Kelvin, it would radiate "heat" away into surrounding space via Hawking radiation, further reducing its mass and raising its temperature. For these two cases, the temperature of the black hole dictates in which direction we would observe a run-away CMB-led growth or run-away evaporation.

The Four Laws of Black Hole Thermodynamics

Zeroth Law of Black Hole Thermodynamics. *The event horizon of a black hole has constant surface gravity κ.*

The event horizon of a black hole is currently defined as the boundary where a photon can no longer escape the gravitational pull of the black hole. By the definition of a boundary, the surface gravity is necessarily constant on this boundary. This law is analogous to the zeroth law of classical thermodynamics, which constrains temperature to be constant through a body in thermal equilibrium. From this, surface gravity can be thought of as being analogous to temperature. The condition that the horizon has constant surface gravity is met for the Reissner-Nordstrom metric, Kerr metric, and Kerr-Newmann metric as solutions to the Einstein and Einstein-Maxwell field equations corresponding to the gravitational field of a black hole.

First Law of Black Hole Thermodynamics. *The change in energy of a black hole is proportional to change in surface area, angular momentum, and electric charge by*

$$dE = \frac{\kappa}{8\pi} dA + \Omega dJ + \Psi dQ \tag{2}$$

Defined by energy E, surface gravity κ, horizon surface area A, angular velocity Ω, angular momentum J, electrostatic potential Ψ, and electric charge Q. Note that for a black hole described by the Reissner-Nordstrom metric, Ω and J will be equal to zero, and for a black hole described by the Kerr metric, Ψ and Q will be equal to zero. Equation 2 comes out elegantly because black hole thermodynamics is expressed in geometrized units, where the speed of light c, gravitational constant G, Coulomb's constant k_e, and electric charge q are all set to unity. This law tells us that while a change in black hole total energy E cannot be directly observed, it is still possible to observe any changes of energy indirectly from the components on the right hand side of Equation 2. This law is analogous to the first law of classical thermodynamics, which describes energy conservation.

Second Law of Black Hole Thermodynamics. *Assuming the weak energy condition, the horizon area is a non-decreasing function of time such that*

$$\frac{dA}{dt} \geqslant 0 \tag{3}$$

This follows from Hawking's Horizon Area Theorem, but was superseded by the discovery of Hawking radiation, as there is the potential for $\frac{dA}{dt} \leqslant 0$ given a sufficiently small black hole, such that $M_{BH} \leqslant M_{moon}$. To reconcile the second law with Hawking radiation, we can draw an analogy from the second law of classical thermodynamics and generalize from there. Recall that the entropy for an isolated system will increase or stay the same for a spontaneous process, then if we include the "outside entropy" of the total system, which includes where ever the Hawking radiation ends up, then we have eliminated the contradiction. From this we have

$$\text{Total Entropy} = \text{Black Hole Entropy} + \text{Outside Entropy} \tag{4}$$

Note that, as above, the weak enegy condition states that for every timelike vector field $\vec{X}$, the matter density ρ observed by the corresonding observers a and b is always non-negative, such that

$$\rho = T_{ab} X^a X^b \geqslant 0 \tag{5}$$

where T_{ab} is the energy-momentum tensor, which describes the distribution of mass, momentum, and stress due to matter within the framework of general relativity.

Third Law of Black Hole Thermodynamics. *It is not possible to form a black hole with vanishing surface gravity, such that $\kappa = 0$ is not possible.*

In order to understand the significance of this law, it is first necessary to draw an analogy to the third law of classical thermodynamics. Recall that the third law states that the entropy of a system in its ground state is well defined and constant. This tells us that a system at absolute zero will have zero entropy. When we first asserted that black holes necessarily have entropy, we made black holes fundamentally incapable with ever reaching absolute zero. Since we know that black holes have non-zero entropy, the ground state $\kappa = 0$ cannot be physically realized.

Defining Black Hole Temperature and Entropy

Following from the above four laws while taking into consideration our current understanding of the quantum mechanical behaviors of systems involving black holes and our use of geometrized units, we find that black holes will emit Hawking radiation with temperature T_H proportional to surface gravity κ as

$$T_H = \frac{\kappa}{2\pi} \tag{6}$$

Following from the first law of black hole thermodynamics, the entropy of the black hole S_{BH} is exactly equal to the surface area of the event horizon A with the constant of proportionality being $\frac{1}{4}$, coming from the classical thermodynamic relationships between energy, temperature, and entropy

$$S_{BH} = \frac{A}{4} \tag{7}$$

Black Holes as Heat Engines

Because black holes have entropy and follow four thermodynamic laws, all of which are analogous to their respective classical thermodynamics counterpart, it is not absurd to consider black holes as heat engines. To do so, it is necessary to look at how it would be possible to extract work from a rotating black hole. The process of doing so would necessitate the use of a black hole's ergosphere - also known as Kerr spacetime - an area outside the event horizon where all objects within the ergosphere will experience frame dragging. Frame dragging within general relativity is a result of spacetime being warped under non-static stationary distributions of mass-energy. Within the context of an ergosphere, frame dragging occurs because a rotating black hole is not perfectly spherical, but oblate around the axis of rotation. This non-uniform shape creates a non-static and stationary (given a reference frame that eliminates black hole space translation) disturbance in space time. Within the ergosphere, an object will necessarily be dragged along with the rotating spacetime. For the case where the object is split in two, there is the possibility that the momentum of the object has been arranged such that half of the object falls into the black hole while the other escapes to infinity. The half of the object that escapes can have greater mass-energy than the original object via transferrence of kinetic energy from the black hole's rotational energy. Through this process, the black hole will see a net decrease in angular momentum, and that decrease corresponds to the energy extracted from the black hole. It should be noted that if this process is repeated enough times, the black hole will lose all of its angular momentum, resulting in it becoming a non-rotating Schwarzschild black hole.

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